Theorem: Finding a NE which maximizes the total payoff is NP-Hard.

Theorem: Finding some NE is PPAD-Complete.

Cubic graphs: degree 3

Theorem of Smith:

Nodes cubic graphs with degree 3 1) adjacent can only
2) adjacent can only
3) adjacent can only
4) adjacent can only
5) adjacent can only

A cubic graph with an Hamilton cycle exists.

If there is no Hamilton cycle, is there no PPCP?

Ar ε-approximate Hamilton lecture: PPAD Complete?

If there is no ε-approximate Hamilton cycle, is there no PPCP?

Proposition: Finding some 1/n - approximate NE is PPAD-Complete.

ε-approximate: \( u_i \cdot s_i \cdot 1, s_i) \geq u_i \cdot (s_i, s_i) - \varepsilon \)

Theorem: There exists a 0.84-approximate polynomial time algorithm.
Theorem: There is an $\varepsilon$-approximate algorithm for NE with running time $O(n \log n)$.

[1] Narakites, Mehta, Lipton

**Algorithm:**

$$A = \begin{bmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{bmatrix}$$

$$B = \begin{bmatrix} y_{11} & \cdots & y_{1m} \\ \vdots & \ddots & \vdots \\ y_{n1} & \cdots & y_{nm} \end{bmatrix}$$

Pure NE? $m^2$ problems: trivial problem

Mixed: Support of player $i$ at a NE = set of strategies with non-zero probability.

Theorem: Every game has a $\varepsilon$-approximate NE with support of size at most $\frac{\log n}{\varepsilon^2}$.

$$A \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} \leq B \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix}$$

$(x, y)$ is NE

Payoff of player 1: $y^TAx$

Payoff of player 2: $y^TBx$

$k$-uniform strategy: we select $k$ strategies (perhaps with repetition) each with probability $\frac{1}{k}$.

Theorem: For every NE $(x^*, y^*)$ and $\varepsilon > 0$, for $k > \frac{12 \log n}{\varepsilon^2}$, there are $k$-uniform strategies $(x, y)$ such that:

$$(x, y): \varepsilon - \text{approximate NE}$$

$$|y^TAx - y^*TAx^*| < \varepsilon$$

$$|y^TBx - y^*TBx^*| < \varepsilon$$
Graphical Games

Find NE: PPAD complete? YES

Correlated equilibria: in P